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On the Synergic Relationships between Special Relativity and Quantum Theories

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The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

The successful results of the relativistic form of a quantum field theory that is derived from a Lagrangian density justify its general usage. The significance of the Euler-Lagrange equations of a quantum particle is analysed. Many advantages of this approach, like abiding by the conservation laws of energy, momentum, angular momentum, and charge are well known. The merits of this approach also include other properties that are still not well known. For example, it is shown that a quantum function of the form $\psi(t, \boldsymbol{r})$ describes a pointlike particle. Furthermore, the Lagrangian density and the Hamiltonian density take a different relativistic form – the Lagrangian density is a Lorentz scalar, whereas the Hamiltonian density is the T^{00} component of the energy-momentum tensor. It is proved that inconsistencies in the electroweak theory stem from negligence of the latter point.

Keywords: Special relativity; quantum fields; the variational principle; Noether theorem.

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1 INTRODUCTION

This work examines the inherent relationships between special relativity and quantum physics. The mathematical structure of these theories plays a fundamental role in the analysis. It is explained how these theories fit each other, and that this issue enhances their credibility. It is further shown that these theories alter several concepts about the structure of the physical world and put restrictions on acceptable quantum theories. An observation of the present literature indicates that some of the results are still not well known. The analysis begins with a brief description of key elements of the 19th century physical theories and concepts. These elements are reexamined in the main part of this work. Mathematical expressions take the standard notation.

Differential equations whose solutions determine the time evolution of the state of a given particle are an important property of the relevant theory. These equations are called the particle's equations of motion. Solutions of the equations of motion of an acceptable physical theory of a given system of particles should be compatible with the particle's experimental data. Below, the existence of differential equations and the fit of their solutions to experimental data are called *the primary properties of a particle's theory.* In particular, it is shown below that a Lagrangian density is the cornerstone of present quantum theories. Hence, the theory's equations of motion are the Euler-Lagrange equations of its Lagrangian density.

Newtonian mechanics successfully describes the motion of macroscopic bodies whose velocity is much smaller than the speed of light. Here a second-order differential equation describes the motion of a particle (see e.g. [1], p. 2)

$$m\frac{d^2\boldsymbol{r}}{dt^2} = \boldsymbol{F}(\boldsymbol{r}, t, \boldsymbol{v}), \tag{1}$$

where $\boldsymbol{F}$ denotes the force. It can be shown that in certain cases Newtonian mechanics can be derived from a function called Lagrangian

$$L(q, \dot{q}, t) = T - V, \tag{2}$$

where q, $\dot{q}$ denote a set of n generalized coordinates and their time-derivatives, respectively. T is the kinetic energy, and V is the potential energy (see e.g. [1], p. 21). Here the equations of motion are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0. \tag{3}$$

This is a set of n second-order differential equations which are the Lagrange equation of the system.

The variational principle pertains to this matter. The action accumulated during the transition of a given system from an initial state at an initial time to a final state at a final time is

$$I = \int_{t_i}^{t_f} L(q, \dot{q}, t)dt. \tag{4}$$

An application of the variational principle to the action of (4) proves that the corresponding Euler-Lagrange equations are the Lagrange equations (3) (see e.g. [1], pp. 44, 45; [2], pp. 2, 3).

An alternative description of the laws of motion relies on the corresponding Hamiltonian. The generalized momentum that is conjugate to the generalized coordinate q_i is (see [1], p. 335; [2], p. 16)

$$p_i = \frac{\partial L}{\partial \dot{q}_i}. \tag{5}$$

The Hamiltonian is a function of the generalized coordinates, their conjugate momenta, and the time. One can apply the Legendre transformation of a given Lagrangian and obtain the required Hamiltonian

$$H(q, p, t) = \sum_i p_i \dot{q}_i - L \tag{6}$$

(see [1], p. 337; [2], p. 131). In the case of n independent coordinates, one derives from the Hamiltonian a set of $2n$ first-order differential equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \tag{7}$$

(see [1], p. 337; [2], p. 132). These equations, which are called the canonical equations, are equivalent to the n second-order Lagrange equations (3).

Two important laws of Newtonian mechanics are:

N.1 The Galilean relativity principle says that the above-mentioned laws of mechanics hold for any frame that moves inertially (see [1], p. 2; [2], pp. 4-6).

N.2 Let $\Sigma,\ \Sigma'$ be two inertial frames and $-\boldsymbol{v}$ denotes the velocity of Σ' with respect to Σ. The coordinates and the time measured in these frames obey these relations

$$\boldsymbol{r}' = \boldsymbol{r} + \boldsymbol{v}t; \quad t' = t \tag{8}$$

(see [1], p. 276; [2], p. 6). It means that time is independent of the relative velocity of inertial frames.

It turns out that standard textbooks on Newtonian mechanics do not discuss wave motion (see e.g. [1, 2]). In classical physics, the wave amplitude $\phi(x,y,z,t)$ denotes a disturbance in the state of a medium. It satisfies the wave equation (see [3], p. 5)

$$\nabla^2\phi = \frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2}, \tag{9}$$

where v denotes the wave velocity. Two elements of the 19th century wave theory are:

W.1 A wave theory regards the existence of a medium as an indispensable element of the wave phenomenon.

W.2 A wave phenomenon is spread over a not very small spatial region.

At the beginning of the 19th century, physicists recognized the wave properties of light. In order to abide by the medium concept W.1, people have postulated the existence of the ether as the medium that carries light waves.

This work shows how special relativity and quantum theories affect these theories and alter human concepts about the structure of the physical world. As a matter of fact, not all results are well known.

Units where $\hbar = c = 1$ are used. Greek indices run from 0 to 3. Most formulas take the standard form of a relativistic covariant expression. The metric is diagonal and its entries are (1,-1,-1,-1). An upper dot denotes the time-derivative. In the above-mentioned units there is one kind of dimension and an appropriate power of the length $[L^n]$ denotes the dimension of a physical quantity. The second section discusses relativistic classical physics. The third section discusses relativistic quantum theories. Results derived from these theories are pointed out in the fourth section. The last section summarizes this work.

2 RELATIVISTIC CLASSICAL PHYSICS

Formulas and equations that describe electromagnetic phenomena were already known in the first half of the 19th century. Maxwell has analyzed these expressions and concluded that they should be modified in order to abide by the charge conservation law. His results are known as Maxwell equations. In the vacuum these equations take the compact tensorial form (see [4], pp. 71, 79; [5], p. 551)

$$F^{\mu\nu}{}_{,\nu} = -4\pi j^\mu, \quad F^{\dagger\mu\nu}{}_{,\nu} = 0, \tag{10}$$

where $F^{\dagger\mu\nu}$ is the dual tensor of the electromagnetic field tensor $F^{\mu\nu}$, and j^μ is the 4-current of the electric charge.

Solutions of Maxwell equations in the vacuum show the existence of transverse electromagnetic waves where the potentials and the fields satisfy the wave equation (9) (see [4], p. 117; (20.8), (20.10) and (20.11) in [6]). An important property of these waves is:

P.1 *Electromagnetic waves travel at the speed of light in every inertial frame.*

About 2 decades later, Hertz carried out an experiment that has shown the existence of electromagnetic waves [7]. Nearly at the same time, Michelson and Morley carried out an experiment aiming to measure the relative velocity of the earth with respect to the inertial frame of the postulated ether. In spite of the earth's motion around the sun, they found a null velocity. This outcome agrees with the Maxwellian result P.1. Hence, this experiment can be regarded as another support of Maxwellian electrodynamics.

The formulation of special relativity puts these issues in a consistent mathematical structure. Here the concept N.1 of Galilean relativity holds but the concept N.2 of absolute time and the space-time transformation between inertial frames (8) are replaced by the Lorentz transformation. If frame Σ' moves in the x-direction with velocity $-v$ with respect to frame Σ then the Lorentz transformation is (see [4], p. 11; [5], p. 516)

$$x' = \frac{x+vt}{\sqrt{1-v^2/c^2}}, \ \ y' = y, \ \ z' = z, \ \ t' = \frac{t+vx/c^2}{\sqrt{1-v^2/c^2}}, \tag{11}$$

where the speed of light c is written explicitly.

An application of the Minkowski space casts relativistic expressions into a neat tensorial form. Thus, (10) are the Maxwell equations in the vacuum and the force exerted on a charge e is the Lorentz force (see [5], p. 551)

$$\frac{dp^\mu}{d\tau} = eF^{\mu\nu}v_\nu, \tag{12}$$

where p^μ, v^ν are the 4-momentum and the 4-velocity of the charged particle, respectively, and τ is the invariant time. The continuity equation represents charge conservation

$$j^\mu_{,\mu} = 0 \tag{13}$$

(see [4], p. 77; [5], p. 549).

Special relativity and Maxwellian electrodynamics have amazing experimental success. For example, the electron's kinetic energy of the LEP collider was about 200,000 times greater than its rest mass, namely $E_k \simeq 200000\, mc^2$ [8]. Here the machine was designed according to the laws of special relativity and the electron's speed has not exceeded the speed of light. Results of Maxwell theory say that the photon is massless and chargeless. The experimental bound of the photon's mass is smaller by a factor of about 10^{-24} times the electronic mass, and the experimental bound of the absolute value of the photon's charge is smaller by a factor of about 10^{-46} times the absolute value of the electronic charge [9]. Furthermore, let ϵ denote the experimental deviation from the Coulomb law (which is embedded in Maxwell equations). Here the electric field of a charge takes the form

$$E = Q/r^{(2+\epsilon)}, \tag{14}$$

and the experimental bound of ϵ is extremely negligible: $|\epsilon| < 10^{-16}$ (see table 2 in [10]).

The extraordinary experimental success of special relativity and Maxwellian electrodynamics is well known. This evidence justifies their application as a basis for the analysis that is presented below.

Two results of special relativity deserve particular attention.

- SR.1 Landau and Lifshitz prove that in the classical domain, the theory of special relativity means that an elementary classical particle is a pointlike object (see [2], pp. 46, 47).
- SR.2 An observation of the low-velocity limit $v \ll c$ of the space-time Lorentz transformation (11) shows that it agrees with the Newtonian transformation (8). This is an example of an important principle concerning the correspondence between physical theories. This principle refers to two coherent theories A, B. If the domain of validity of theory A is included inside the domain of validity of theory B then an expression of theory A should agree with the limit of an appropriate expression of theory B.

The present literature recognizes the correspondence principle SR.2 in several cases. Besides the above-mentioned correspondence between relativistic mechanics and Newtonian mechanics, one can find in the literature correspondence relationships between other theories. Two examples are relevant to the analysis presented below. The success of classical mechanics means that "classical mechanics must therefore be a limiting case of quantum mechanics." (see [11], p. 84; [12], pp. 25-27, 137, 138). Furthermore, quantum field theory (QFT) corresponds to quantum mechanics. For example, a well-known textbook states explicitly: "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear and condensed matter physics" (see [13], p. 49). This statement means that there are certain relationships between QFT and quantum mechanics. The combined meaning of these quotations is that QFT corresponds to classical physics. One can find on pp. 3-6 of [14] a general discussion of the correspondence between physical theories.

A special topic of the foregoing issues is that the correspondence between quantum theories and classical physics says that the classical physics pointlike attribute of an elementary particle of item SR.1 should also apply to quantum theories. It is interesting to see how the standard structure of quantum theories coherently combines the apparently inconsistent pointlike attribute of an elementary particle with its spatially distributed wave properties.

The foregoing arguments yield another requirement that a quantum theory must satisfy. Solutions of the equations of classical physics (1) or (7) determine the position of a classical particle as a function of the time. Hence, a quantum theory must provide an expression whose limit agrees with the particle's position.

3 RELATIVISTIC QUANTUM THEORIES

Two principles belong to the basis of a quantum theory.

QT.1 De Broglie postulated in 1924 that a quantum particle has wave properties that are described by a function whose undulating factor is

$$\Phi = e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}. \tag{15}$$

Here $\boldsymbol{k}$ takes the value of the particle's momentum $\boldsymbol{p}$, and ω is its energy (see [11], pp. 119, 120; [12], p. 3). A few years later, experiments with the electron confirmed this principle. At present experiments have already confirmed wave properties of elementary particles and of some composite particles as well (see [15]).

This principle abides by relativistic requirements. Indeed, the power series expansion of the exponent of (15) is

$$e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} = 1 + i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t) + ... \tag{16}$$

Here the pure number 1 is a dimensionless Lorentz scalar. Therefore, $(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)$ should also be a dimensionless Lorentz scalar. (The Lorentz scalar of the phase ensures the obvious requirement saying that the same interference pattern is seen in every inertial frame.) The relativistic scalar product of the space-time coordinates 4-vector $(t, \boldsymbol{x})$ and the particle's energy-momentum 4-vector $(E, \boldsymbol{p})$ provides the required expression. Hence, the de Broglie principle determines the identification of the wave properties $(\omega, \boldsymbol{k})$ with the particle's energy and momentum $(E, \boldsymbol{p})$.

QT.2 A few years later Heisenberg published the uncertainty principle which says that inherent quantum properties prevent a simultaneous accurate measurement of conjugate quantum quantities. for example, let Δ denote the uncertainty of a quantum variable, then

$$\Delta x \cdot \Delta p_x \geq \hbar, \;\; \Delta t \cdot \Delta E \geq \hbar \tag{17}$$

(see [12], p. 7).

A very short-lived particle has a quite small time-uncertainty. The uncertainty relations (17) say that this particle should have a quite large energy uncertainty. And indeed, one finds that the uncertainty of the experimental energy of such a particle provides a clear demonstration of this principle. For example, the quite large energy uncertainty of the $\Delta^{++}(1232)$ baryon that is seen in the $\pi^+ p$ cross section data is a convincing example of this issue (see [16], p. 131). This is an example of a physical law, and the data of every short-lived particle show the width of its mass/energy cross section [9].

The uncertainty relations (17) provide a convincing reason that explains why a quantum theory cannot take the mathematical structure of classical physics. Thus, in classical physics, the equations of motion (3) are derived from a Lagrangian. The Legendre transformation yields the Hamiltonian and its canonical equations (7), which are equivalent to (3). The canonical equations use the Hamiltonian (6) where the coordinates and their conjugate momenta play the role of *independent variables*. As such, these variables and their derivatives should be known accurately. This requirement cannot be reconciled with the uncertainty relations (17).

At present, the generally accepted structure of quantum theories uses the quantum function $\psi(x)$ and its derivatives as independent variables. Here x denotes the four space-time coordinates and the action takes the form

$$I(\psi) = \int d^4x \mathcal{L}(\psi, \psi_{,\mu}), \tag{18}$$

where $\mathcal{L}(\psi, \psi_{,\mu})$ is a Lagrangian density. This Lagrangian density $\mathcal{L}(\psi, \psi_{,\mu})$ and the action

$I(\psi)$ are mathematically real Lorentz scalars. Mainstream textbooks use this approach. For example, a well-known textbook supports this approach and states: "all field theories used in current theories of elementary particles have Lagrangians of this form" (see [13], p. 300). An application of the variational principle to (18) yields the Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial x^\mu}\frac{\partial \mathcal{L}}{\partial(\partial\psi/\partial x^\mu)} = 0 \qquad (19)$$

(see [13], p. 300). For every given quantum theory, these equations are the particle's equations of motion. This approach is also adopted by the present work.

It is explained below why the Noether theorem is an important element of this structure of quantum theories. This theorem connects between symmetries of the Lagrangian density and conservation laws that the theory abides with. Let us see how this theoretical structure satisfies self-evident quantum requirements and imposes constraints on acceptable quantum theories.

REQ.1 The Euler-Lagrange equations of the action principle (18) are the differential equations of the system. Hence, the required equations of motion take an explicit form. If solutions of these equations adequately describe experimental data then the primary requirement of a particle's theory are satisfied.

REQ.2 A Lagrangian density that takes the form $\mathcal{L}(\psi, \psi_{,\mu})$ does not depend explicitly on the space-time coordinates $(t, \boldsymbol{x})$. Hence, the Noether theorem assures that the theory conserves energy-momentum and angular momentum (see [17], pp. 17-19).

REQ.3 The Lagrangian density $\mathcal{L}(\psi, \psi_{,\mu})$ is a Lorentz scalar. Hence, the Noether theorem assures that the theory takes a relativistic covariant form (see [17], pp. 17-19).

REQ.4 If the Lagrangian density $\mathcal{L}(\psi, \psi_{,\mu})$ is a homogeneous quadratic function of $\psi(x)$ then the Euler-Lagrange equation (19) takes a linear homegeneous form with respect to the quantum function $\psi(x)$. This is the form of the wave equation (see [3], p. 5; [12], chapter II). It is well known that the literature contains examples of quantum theories of specific particles whose function $\psi(x)$ satisfies the wave equation.

REQ.5 The function $\psi(x)$ $(x \equiv (x, y, z, t))$ describes a point-like elementary particle. Indeed, the independent variable x can describe the probability of finding the particle at x but it cannot describe how a composite particle is distributed at the vicinity of x. For this reason, a quantum theory that is based on (18) satisfies the required correspondence to the classical theory, where item SR.1 shows that an elementary classical particle is pointlike.

REQ.6 The Noether theorem provides a prescription for constructing a conserved 4-current for a quantum theory of the form (18). The invariance of a quantum field Lagrangian density under a global phase transformation

$$\psi \rightarrow \exp(i\alpha)\psi, \qquad (20)$$

where α is a mathematically real variable, yields

$$0 = i\alpha\Big[\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu\Big(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\Big)\Big]\psi + i\alpha\partial_\mu\Big(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\psi\Big) \qquad (21)$$

(see [18], p. 314). The Euler-Lagrange equation (19) proves that the quantity enclosed inside the square brackets vanishes. Since α does not vanish identically, one finds that the expression that is written inside the last bracket of (21) is a conserved 4-current

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\psi. \qquad (22)$$

The action (18) is a mathematically real quantity. Therefore, the invariance of the Lagrangian density $\mathcal{L}(\psi, \psi_{,\mu})$ of (18) under a global phase transformation is obtained if each of its terms takes the form $\psi^\dagger \hat{O}\psi$, where $\hat{O}$ is a Hermitian operator.

The 0-component of a conserved 4-current j^0 is the density $\rho(t, \boldsymbol{x})$ of the quantum particle (see [2], p. 75). Let V denote a spatial volume. It means that at time t, the probability P of finding the quantum particle at V is

$$P = \int_V \rho(t, \boldsymbol{x}) d^3x. \tag{23}$$

Therefore, the classical limit of (23) describes the particle's position, which is required by the correspondence principle. Hence, *an acceptable quantum theory should provide a coherent expression for density.*

This quite long list of favorable properties of a quantum theory that are derived from the action of (18) explains why QFT textbooks use it as a basis for the analysis. Further properties of this theoretical structure, where some of which are still not very well known, are discussed in the next section.

4 DISCUSSION

Humans can see how a wave propagates in a pond. They can also "feel" how an acoustic wave travels through a gas, a liquid or a solid. These are examples of a wave that is a disturbance of the state of a medium. However, this kind of experiments *cannot* rule out the existence of a wave phenomenon that is independent of a medium. Indeed, in physics, a wave is a physical object that satisfies the wave equation (9). Here the medium is not an explicit part of the equation. Referring to electromagnetic waves, the ether concept as a medium that carries these waves has been abandoned after the rise of special relativity. The confirmation of the de Broglie hypothesis of the wave nature of a quantum particle adds another blow to the ether concept. This issue means that the photon's ether concept requires the addition of a different kind of ether for every kind of a quantum object, like the electron, the muon, and even the neutron. This is a quite weird combination of several kinds of ether which has no support in the scientific literature.

As a matter of fact, physics does not rely on concepts that depend on human senses, but on a coherent mathematical structure that adequately describes experimental data. For example, item REQ.4 of the previous section shows that the quantum function $\psi(x)$ satisfies the wave equation, and the ether concept is not a part of the theory. This issue substantiates the concept where the wave phenomenon is an inherent property of a quantum particle.

Furthermore, item REQ.5 shows that an elementary quantum particle has pointlike attributes. Experiments support the pointlike properties of an elementary particle. Indeed, the electron is the best known elementary particle, and the experimental upper bound of its radius is $r_e < 10^{-20}\, cm$ [19]. This figure is 7 orders of magnitude smaller than the proton radius. Here the coherent theoretical structure prevails, and the apparently problematic aspects of the human concept concerning the wave/particle dilemma has no scientific merit.

It turns out that the pointlike attribute of an elementary particle is still not very well-known. For example, an article that has been cited hundreds of times uses the π, ρ mesons as carriers of the nuclear force [20]. However, it is well known that these mesons are quark-antiquark bound states (see [21], p. 222) and their self-volume does not vanish. As such, the meson wave function takes the form $\phi(\boldsymbol{r}_1, \boldsymbol{r}_2, t)$, where $\boldsymbol{r}_i$ denotes the quark-antiquark coordinates, respectively. On the other hand, the quantum function of an interaction mediating particle $\phi(x)$ pertains to a point-like particle. Therefore, the π, ρ mesons cannot be regarded as interaction mediating particles and the basis of [20] collapses.

Let us examine another issue. The laws of motion of a classical particle are derived from a Lagrangian (2) or, alternatively, from a Hamiltonian (6). It is shown above that the theory of a quantum particle uses a Lagrangian *density*. This issue means that an appropriate Legendre transformation of the Lagrangian density yields a Hamiltonian density. The energy-momentum tensor is obtained from a given Lagrangian density

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\psi_{,\nu})} g^{\alpha\mu} \psi_{,\alpha} - g^{\mu\nu} \mathcal{L} \tag{24}$$

(see [22], p. 310). The component T^{00} of this tensor is the Hamiltonian density

$$\mathcal{H} = T^{00} = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} \dot{\psi} - \mathcal{L} \tag{25}$$

(see [17], p. 16).

These expressions uncover the different relativistic structure of the Lagrangian density of (18), which is a Lorentz scalar, and that of the Hamiltonian density (25), which is the T^{00} component of a second rank tensor. This distinction has far-reaching consequences. An important example is the term of the tensor interaction of a Dirac particle called the Pauli term

$$\mathcal{L}_{int} = \mathrm{d}\bar{\psi}\sigma_{\mu\nu}\mathcal{F}^{\mu\nu}\psi \tag{26}$$

(see[23]; [13], p. 14). This term is sometimes called *tensor interaction* because of its $\sigma_{\mu\nu}$ factor (see [24], p. 26). The Pauli term (26) is a Lorentz scalar which belongs to a Lagrangian density. An examination of the Hamiltonian density shows that the corresponding term of (26) boils down to the difference between two terms, a vector and an axial vector [25]

$$\mathcal{H}_{int} = 2\mathrm{d}\psi^{\dagger}(i\gamma_i\mathcal{E}^i - \gamma^5\gamma_i\mathcal{B}^i)\psi \tag{27}$$

This term agrees with the V-A (Vector-Axial vector) attribute of weak interactions (see [16], pp. 214-220).

The historical development of the electroweak theory demonstrates the significance of this issue. Thus, [26] is a key article in the development of the electroweak theory. The authors of [26] examine the Lagrangian density of the problem and reject the tensor interaction T as a possible candidate (see the text above their (8)). Their work does not distinguish between relativistic properties of a Lagrangian density and those of a Hamiltonian density. In order to account for the parity violation feature of weak interactions, they introduced the parity violation factor $(1 \pm \gamma^5)$. This factor has only two degrees of freedom. Hence, it implies a two-component massless neutrino. As a matter of fact, the concept of a massless neutrino has become an element of the electroweak sector of the standard model. Several references substantiate this claim: The electroweak theory relies on "a neutrino which travels exactly with the velocity of light" [27]. A review article restates the neutrino masslessness attribute of the electroweak theory: "Two-component left-handed massless neutrino fields play a crucial role in the determination of the charged current structure of the Standard Model" (see the Abstract of [28]). Similarly, a textbook states: "Neutrino masses are exactly zero in the Standard Model" (see [29], p. 533).

As stated above, the arguments of [26] ignore the distinction between relativistic attributes of a Lagrangian density and those of a Hamiltonian density. It means that this article and the associated electroweak theory rely on an erroneous basis. Here are several points that support this claim.

EW.1 Contrary to the neutrino masslessness feature of the electroweak theory, experimental data show that the neutrino is a massive particle [30].

EW.2 As stated above, the time-evolution of a particle is described by differential equations called the equations of motion of the particle. The equations of motion of a quantum particle are the Euler-Lagrange equations of its Lagrangian density. Section 3 shows arguments that explain why presently accepted theories use a Lagrangian density as the basis of their mathematical structure. Primary properties of a particle's theory say that consistent equations of motion whose solutions adequately fit relevant experimental data are vital for an acceptable quantum theory. For example, it is well known that every textbook on the Dirac theory of a spin-1/2 massive particle shows the explicit form of the Dirac equation, which is a partial differential equation. Some textbooks also show specific solutions of this equation that adequately fit experimental data. By contrast, no textbook shows the explicit form of the differential equations that are the equations of motion of the electroweak particles. A fortiori, no solution of the omitted equations is tested with respect to experimental data. It means that the electroweak theory is inconsistent with the primary properties of a particle's theory.

EW.3 An analogous problem is the fact that no QFT textbook shows an explicit expression for the density of the electroweak Z particle. As explained above, item REQ.6 of the previous section explains why this is a failure to show a vital theoretical quantity.

EW.4 In quantum mechanics, the eigenvalue of a Hermitian operator is (see e.g. [12], p. 47)

$$<O>=\int \psi^* \hat{O} \psi d^3x, \qquad (28)$$

where $< O >$ denotes the eigenvalue of the variable that is represented by the operator $\hat{O}$. Evidently, the dimension of an operator is the same as that of its eigenvalue. Hence, in quantum mechanics, the dimension of the wave function ψ is $[L^{-3/2}]$. Considering this aspect, one finds that the first order-Dirac theory abides by the Weinberg correspondence principle. Second-order quantum equations, like that of the electroweak theory, do not share this property because the dimension of the product of their functions $\phi^\dagger\phi$ is $[L^{-2}]$.

EW.5 Many other electroweak discrepancies are discussed elsewhere (see e.g. [31] and references therein).

It is quite strange to find that the mainstream literature does not mention that the electroweak theoretical structure lacks these vital elements. For example, they are not mentioned in the List of Unsolved Problems in Physics [32].

5 CONCLUSIONS

Merits of the relativistic covariant form of QFT are discussed. Special relativity and quantum theories have been constructed in the 20th century, and they have introduced a profound change of the scientific concept concerning the structure of the physical world. In particular, the concepts of universal time, the need for a medium that carries a wave motion, the conflicting notions of a (pointlike) particle and a wave, and the strict determinism of a physical process have been abandoned.

It is now recognized that the relativistic form of a QFT and the variational principle are the basis of quantum theories. Here section 3 explains how an appropriate Lagrangian density of a quantum theory (18) ensures that the theory abides by several physical requirements. Quite a few of these issues are well known, but there are still some important quantum features that deserve a further discussion. In particular, the paper proves that the quantum function $\psi(t, \boldsymbol{r})$ describes a pointlike particle. Another important result is the different covariant properties of the Lagrangian density, which is a Lorentz scalar, and the Hamiltonian density, which is the T^{00} component of the energy-momentum tensor. This distinction has far-reaching consequences. In particular, it shows that the Standard Model electroweak theory is based on an erroneous concept. Here the tensor term T of the Lagrangian density boils down to V-A terms of the Hamiltonian density. These terms are in accordance with well known weak interaction data. Other drawbacks of the electroweak theory support this conclusion (see e.g. [31] and references therein).

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

[1] Goldstein H, Poole C, Safko J. Classical mechanics. 3rd Edition. Addison Wesley, San Francisco; 2002.

[2] Landau LD, Lifshitz EM. Mechanics. Pergamon, Oxford; 1960.

[3] Coulson CA. Waves. Oliver and Boyd, Edinburgh; 1961.

[4] Landau LD, Lifshitz EM. The classical theory of fields. Elsevier, Amsterdam; 2005.

[5] Jackson JD. Classical electrodynamics. New York: John Wiley; 1975.

[6] Feynman RP, Leighton RB, Sands M. The Feynman lectures on physics. V. II, Addison-Wesley, Reading Mass; 1965.

[7] Avalable:https://en.wikipedia.org/wiki/Heinrich_Hertz

[8] Avalable:https://home.cern/science/accelerators/large-electron-positron-collider

[9] Tanabashi M, et al. Particle data group. Phys. Rev. D. 2018;98:030001. Avalable:http://pdg.lbl.gov/2019/listings/rpp-2019-list-photon.pdf

[10] Tu LC, Luo J. Metrologia. 2004;41:S136 Avalable:https://www.researchgate.net/publication/228689980

[11] Dirac PAM. The principles of quantum mechanics. Oxford University Press, London; 1958.

[12] Schiff LI. Quantum mechanics. McGraw-Hill, New York; 1955.

[13] Weinberg S. The quantum theory of fields. Vol. I. Cambridge University Press, Cambridge; 1995.

[14] Rohrlich F, Classical charged particle. Third Edition, World Scientific, Singapore; 2007.

[15] Avalable:https://en.wikipedia.org/wiki/Double-slit_experimentInterference_of_individua_particles

[16] Perkins DH. Introduction to high energy physics. Menlo Park CA, Addison-Wesley; 1987.

[17] Bjorken JD, Drell SD. Relativistic quantum Fields. McGraw-Hill, New York; 1965.

[18] Halzen F, Martin AD. Quarks and leptons, an introductory course in modern particle physics. John Wiley, New York; 1984.

[19] Dehmelt H. Physica scripta. 1988;T22:102.

[20] Ishii N, Aoki, Hatsuda T. Phys. Rev. Lett. 2007;99:022001.

[21] Thomson M. Modern particle physics. Cambridge University Press, Cambridge; 2013.

[22] Peskin ME, Schroeder DV. An introduction to quantum field theory. Addison-Wesley, Reading Mass; 1995.

[23] Pauli W. Relativistic field theories of elementary particles. Rev. Mod. Phys. 1941;13:203-232.

[24] Bjorken JD, Drell SD. Relativistic quantum Mechanics. McGraw-Hill, New York; 1964.

[25] Comay E. A theory of weak interaction dynamics. Open Access Library Journal. 2016;3:1-10. Avalable:https://www.scirp.org/journal/PaperInformation.aspx?paperID=72788

[26] Feynman RP, Gell-Mann M. Phys. Rev. 1958;109:193.

[27] Salam A. Nobel lecture. Avalable:https://www.nobelprize.org/uploads/2018/06/salam-lecture.pdf

[28] Bilenky SM. Phys. Part. Nuclei. 2015;46:475.

[29] Srednicki M. Quantum field theory. Cambridge University Press, Cambridge; 2007.

[30] Formaggio JA, Zeller GP. Rev. Mod. Phys. 2012;84:1307.

[31] Comay E. Differences between two weak interaction theories. Phys. Sci. Int. J. 2019;21(1):1. Avalable:http://www.journalpsij.com/index.php/PSIJ/article/view/30091/56456

[32] Avalable:https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_physics

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