



Fluctuational Dynamics of Extended Systems: Activation-Tunnel Frontier

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

The dynamics of systems in barrier structures is determined by the rate of the fluctuational decay of metastable states in a potential relief. The nature of the decay undergoes a qualitative change with a variation of the temperature. As the temperature decreases, thermal fluctuations freeze out and are replaced by quantum ones, which leads to a kind of phase transition in the dynamics. The transition temperature depends on the degree of metastability and can be controlled by an external load. This dependence is calculated for an extended nanosystem in an inclined periodic relief of the "washboard" type in a wide range of load changes. The obtained dependence generalizes the previously known results and can serve as the phase diagram of various dynamics mechanisms.

Keywords: Dynamics of extended systems; metastable states; slip phase; activation-tunneling transition; collective coordinates.

1. INTRODUCTION

The dynamics of extended quasi-one-dimensional objects of various nature is currently being actively studied both experimentally and

theoretically [1,2]. Popular examples are the motion of charge-density waves, the phenomenon of phase slip in Josephson junctions or superconducting nanowires (see, for example, [2,3]). Similar features are also

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observed in the dynamics of domain boundaries [4] or dislocation movement in the crystal relief [5]. The tendency to miniaturize devices has led to workspaces reaching sizes where the fundamental quantum mechanisms come into play. Therefore, the study of phenomena on the so-called "classical / quantum frontier" becomes very relevant. The distinction between classical and quantum mechanisms is all the more urgent in view of the emergence of such purely quantum applications of nanosystems as storing information for quantum computers in the form of qubits, quantum teleportation, distribution of quantum keys, and others [6,7].

The fundamental characteristic in the low-temperature physics is the boundary that separates the temperature regions of classical thermally activated and quantum mechanical dynamics of systems [8].

Of considerable interest for applications is the reaction of an extended system located in a flat bistable or periodic potential relief to the impact of an external load making the relief valleys nonequivalent. The position of such the system in a minimum of the potential relief disturbed by the load becomes a metastable state, since, overcoming the barrier by means of thermal or quantum fluctuations, the object can move to neighboring energetically preferred minima. For systems of recording and storing information in barrier structures of spintronics, the issues of switching and stability of states in minima of the potential relief are important [9].

According to the accepted concepts, the evolution of a sufficiently extended system is carried out by the formation of local nuclei of a new state, their expansion and merging. The kinetics of this process is often described in terms of the formation and movement of nucleus boundaries, which are domain walls or kink-solitons (hereinafter referred to as kinks) [4]. With regard to Josephson junctions, the terms fluxon or anti-fluxon are sometimes used instead of kink or anti-kink.

The dynamics of domain walls or kinks is well studied for relatively low loads, at which they can be considered as weakly perturbed quasiparticles characterized by a single degree of freedom – the position of the kink as a whole [10]. The situation is more complicated when the load increases, leading to deformation of the kinks, which reveals their internal degrees of freedom. In this case, the possibility of one-

dimensional description is lost and it is necessary to use more complete representations of the configuration space of systems. The purpose of this work is to describe the dynamics of extended metastable systems in a wider range of loads, for which an effective method of collective coordinates will be applied. This method includes an additional variable – the width of the domain wall that can change and effectively takes into account the internal degrees of freedom. The foundations of this approach were laid in [11,12].

2. ENERGY RELIEF IN MANYDIMENTIONAL SPACE

A local overcoming barriers by an extended system occurs with a distortion of its configuration. To describe this process, one needs to know the energetics of the configuration space. The archetypal model used to describe the switching of states of various quasi-one-dimensional systems is the elastic string model. The energy of a string located in a flat potential relief $U_0(y)$ under the action of an external force f is described by the expression

$$E\{y(x,t)\} = \int_{-\infty}^{\infty} dx \left\{ \frac{\rho}{2} \left(\frac{\partial y}{\partial t} \right)^2 + \frac{\kappa}{2} \left(\frac{\partial y}{\partial x} \right)^2 + U_0(y) - f y(x,t) \right\}. \quad (1)$$

Here $y(x,t)$ is the string configuration, x is the coordinate along the valleys of the potential $U_0(y)$, which has several minima corresponding to the metastable states of the system, κ is the string stiffness, ρ is the mass density per unit length. The role of the external force in different systems is played by different physical quantities. For example, in the dynamics of charge density waves this is an electric field, in superconductors it is the flowing current or the magnetic field, and at dislocations movement, it is the mechanical stress. The simplest and most popular choice for the potential $U_0(y)$ is the harmonic one

$$U_0(y) = \frac{U_m}{2} \left[1 - \cos\left(\frac{2\pi}{h} y\right) \right]. \quad (2)$$

Here h is the period of the potential. In this case and in the absence of an external load f the Euler-Lagrange equation for the model (1) is the famous sine-Gordon equation, which describes nonlinear waves, in particular, kink-solitons.

The sine-Gordon equation arises in a very diverse range of applications (see for review [13]). These started as early as the 1860s when

it was discovered in the course of an investigation of surfaces of constant negative curvature. It acquired renewed interest due to the classical study of Frenkel and Kontorova in the 1930s in the theory of crystal dislocations. Subsequently, the work of A.C. Scott produced a mechanical analog of the system, through the realization of an array of coupled torsion pendula that proved extremely useful in the experimental observation of its solutions. The range of relevant applications kept expanding through the emergence of Josephson junction arrays and their fluxons, as well as breathers that were intensely studied in the 1990s and early 2000s. Relevant applications of the model have continued to expand with recent proposals involving among others the astrophysics or the evolution of the electromagnetic field on neuronal microtubules [4,13].

The availability of exact solutions to the sine-Gordon equation has caused a boom in related research. However, in the presence of an external load, the integrability property of the model is lost. For this reason, the behavior of systems under low load has mainly been studied, when the perturbation theory is applicable [10]. To study the behavior of a system at any value of the load, one needs to go beyond the close scope of the perturbation theory. This is done in this work using the method of collective coordinates.

In the presence of an external driving force f the metastable states of the system in minima of $U_0(y)$ have a finite lifetime. The decay time of the metastable states is determined by the height of the barriers in the configuration space, and is calculated using expression (1).

We will measure y in units of h , x in $d_0 = h(\kappa/U_m)^{1/2}$, and time t in units of $h(\rho/U_m)^{1/2}$. The energy of the string with potential (2) takes the form

$$E(y(x,t)) = \int_{-\infty}^{\infty} dx \left\{ \frac{1}{2} \left(\frac{\partial y}{\partial t} \right)^2 + \frac{1}{2} \left(\frac{\partial y}{\partial x} \right)^2 + U_0(y) - fy(x,t) \right\} \quad (3)$$

where the variables are dimensionless: $E \rightarrow E/h\sqrt{\kappa U_m}$, $f \rightarrow fh/U_m$. Let the string initially be at the minimum of the potential $U_0(y)-fy$ corresponding to $y_0 = \arcsin(f/\pi)$. Next, we count $y(x)$ from y_0 and renormalize the potential $U_0(y)-fy$, so that the minimum will correspond to zero energy

$$U(y) \rightarrow U(y) = U_0(y) - fy - U_0(y_0) + fy_0 = \frac{1}{2} \{ [1 - (f/\pi)^2]^{1/2} [1 - \cos(2\pi y)] + (f/2\pi) \sin(2\pi y) \} - fy.$$

Here y is counted from 0.

The main task is to describe the process of formation of a new state nucleus corresponding to the easiest path to overcome the barrier. This path will be sought by the variational method, using a trial function to describe the configuration of the string, which depends on 2 parameters: d and x_0

$$y(x) = y_0 + \frac{\exp[(x+x_0)/d]}{\{1 + \exp[(x-x_0)/d]\} \{1 + \exp[(x+x_0)/d]\}} \quad (4)$$

The physical meaning of the parameters becomes clear in the limit $x_0 \rightarrow \infty$, when

$$y(x,t) \rightarrow y_0 + \frac{1}{1 + \exp[(x-x_0)/d]} - \frac{1}{1 + \exp[(x+x_0)/d]},$$

which looks like a kink-antikink pair (see Fig. 1). In this case, $x_0/2$ corresponds to the distance between the kinks, and d to the width of the kink. Taking into account this interpretation, for the sake of convenience, x_0 will be called the longitudinal coordinate, and d - the transverse one. The variables x_0 and d can be time dependent. For small or negative x_0 , corresponding to small deviations of the string from the initial position, the clear meaning of the variables is lost, and x_0, d are considered only as variational parameters. The chosen trial function (4) makes it possible to describe a wide spectrum of intermediate states from small sub-barrier fluctuations to fully formed nuclei behind the barrier.

We substitute trial function (4) into energy functional (3) to obtain the two-dimensional potential relief $E(x_0, d)$. After a change of the scale of the integration variable and replacement the variable x_0 by $e = \exp(x_0/d)$, the potential takes the form

$$E(e, d) = \frac{e^2}{2d} I_1 + \frac{d}{2} \{ [1 - (f/\pi)^2]^{1/2} I_2 - f [2e I_3 - (1/\pi) I_{22}] \}. \quad (5)$$

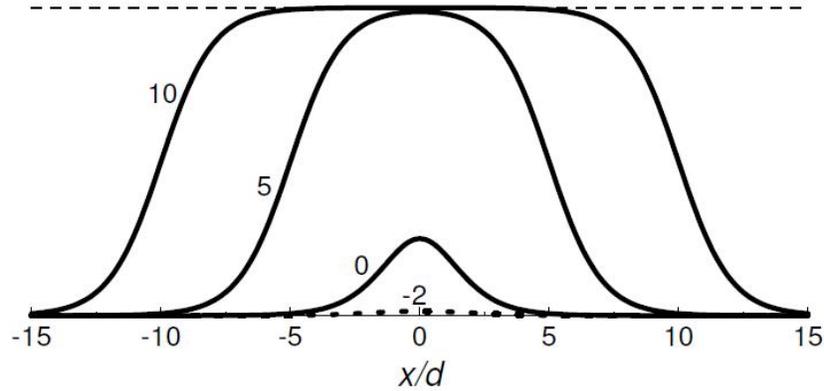


Fig. 1. The form of the trial function $y(x)-y_0$ for different values of the parameter x_0/d , indicated by numbers at the curves. The dotted line shows the trial function with a negative value of the parameter $x_0/d=-2$. The dashed line corresponds to the minimum of the potential relief

Here I_1 , I_2 , I_{22} and I_3 are functions of the single variable e , represented by integrals, for some of which it is possible to obtain analytical expressions, while the rest must be calculated numerically:

$$I_1 = \int_{-\infty}^{\infty} dx \frac{\exp(2x)[1 - \exp(2x)]^2}{[1 + (e+1/e)\exp(x) + \exp(2x)]^4}$$

$$= \frac{1}{e^2} \left(\frac{e^2}{e^2-1} \right)^2 \left\{ \frac{1}{3} + \frac{4e^2}{(e^2-1)^2} - 4 \frac{e^4 + e^2}{(e^2-1)^3} \ln(e) \right\}. \quad (6)$$

$$I_2 = \int_{-\infty}^{\infty} dx \left\{ 1 - \cos \left[2\pi e \frac{\exp(x)}{1 + (e+1/e)\exp(x) + \exp(2x)} \right] \right\}. \quad (7)$$

$$I_{22} = \int_{-\infty}^{\infty} dx \sin \left[2\pi e \frac{\exp(x)}{1 + (e+1/e)\exp(x) + \exp(2x)} \right]. \quad (8)$$

$$I_3 = \int_{-\infty}^{\infty} dx \frac{\exp(x)}{1 + (e+1/e)\exp(x) + \exp(2x)}$$

$$= \frac{1}{\sqrt{e^2-4}} \ln \frac{e + \sqrt{e^2-4}}{e - \sqrt{e^2-4}}. \quad (9)$$

In order to find the optimal way to overcome the barrier, the minimum $E(e,d)$ in (5) with respect to d is calculated with the help of the equation

$$\frac{\partial E}{\partial d} = -\frac{1}{2d^2} e^2 I_1 + \frac{1}{2} \{ [1 - (f/\pi)^2]^{1/2} I_2 - f [2eI_3 - (1/\pi)I_{22}] \} = 0.$$

Whence the optimal d is

$$d = \left[\frac{e^2 I_1}{[1 - (f/\pi)^2]^{1/2} I_2 - f [2eI_3 - (1/\pi)I_{22}]} \right]^{1/2}. \quad (10)$$

Substituting this value into $E(e,d)$ (5), one find the change of energy along the e coordinate trough the valley of the two-dimensional relief

$$E_v(e) = e \{ I_1 \{ [1 - (f/\pi)^2]^{1/2} I_2 - f [2eI_3 - (1/\pi)I_{22}] \} \}^{1/2}. \quad (11)$$

This line will be called the valley bottom of the potential (two-dimensional) relief. When the longitudinal coordinate changes along the valley, the maximum energy E_M is encountered. From the point of view of the two-dimensional relief, this point corresponds to a pass or "saddle" with a decrease in energy when moving away from it along one coordinate and an increase in energy along the other. Fig. 2 illustrates the two dimensional potential relief for the driving force $f=1$ by depicting the lines of a constant energy, and the valley bottom line leading to the pass point in the relief. Note that E_M plays the role of activation energy at the thermally fluctuational formation of the nucleus of the new state of the system.

For a demonstration of the effectiveness of the collective coordinates method used, in Fig. 3 the obtained dependence of the energy E_M on the driving force f is compared with the same dependence calculated numerically in frame of the full multidimensional approach.

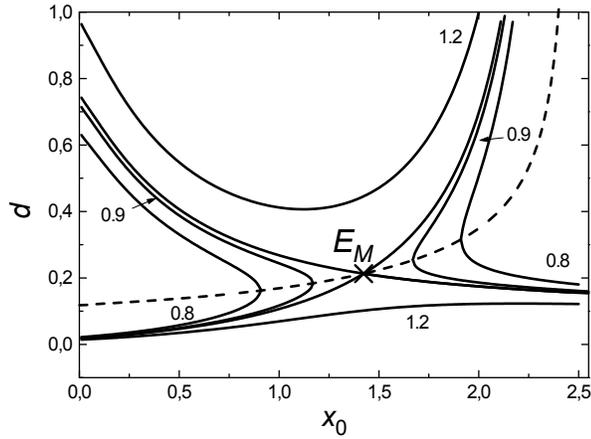


Fig. 2. Two-dimensional potential relief for the driving force $f=1$. Solid lines correspond to the constant energy levels with the values indicated by numbers near them, the dashed line represents the bottom of the potential relief valley. The cross marks the position of the barrier maximum $E_M = 0.93389$

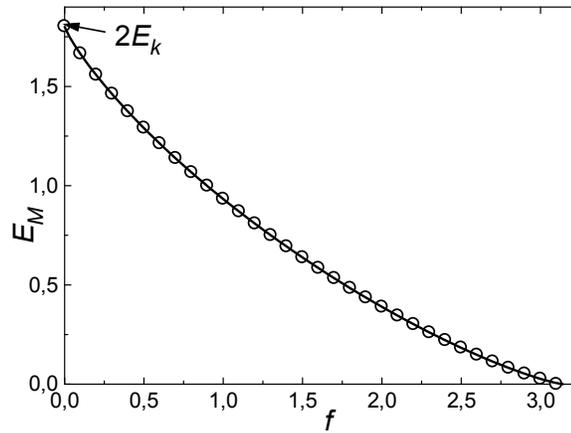


Fig. 3. The dependence of the energy E_M of the formation of the new state nucleus on the driving force f , obtained using two collective coordinates (circles). The solid line shows the result of numerical calculation of this dependence with the full multidimensional approach. E_k is the energy of a kink

The method that takes into account a few, in the simplest version, two collective coordinates, makes it possible to find also the dependence of the quantum tunneling rate on the driving force in a wide range of its variation.

3. QUANTUM MECHANISM OF DECAY OF THE METASTABLE STATE OF AN EXTENDED SYSTEM

In the semiclassical approximation, the probability of quantum-mechanical overcoming

the barrier is given with exponential accuracy by the expression

$$\Gamma \approx \exp(-S/\hbar), \tag{12}$$

where S is the action calculated along the trajectory of the sub-barrier transition, \hbar is the Planck constant. Substitute trial function (4) into (14), assuming x_0 and d to be time dependent

$$y(x,t) = y_1 + \frac{\exp[(x+x_0(t))/d(t)]}{1 + \{\exp[x_0(t)/d(t)] + \exp[-x_0(t)/d(t)]\} \exp[x/d(t)] + \exp[2x/d(t)]}$$

The kinetic energy takes the form

$$T_k = \frac{e^2}{2d} I_{kx} \dot{x}_0^2 + \frac{e^2}{2d} I_{kd} \dot{d}^2 + \frac{e^2}{d} I_{kxd} \dot{x}_0 \dot{d}, \quad (13)$$

Where

$$I_{kx} = \int_{-\infty}^{\infty} dx \exp(2x) \frac{[1 + \exp(2x) + 2e^{-1} \exp(x)]^2}{[1 + (e+1/e)\exp(x) + \exp(2x)]^4} = \frac{1}{3} \frac{e^2}{(e^2-1)^2} - \frac{8e^4 + 12e^2}{(e^2-1)^4} + \frac{4e^6 + 28e^4 + 8e^2}{(e^2-1)^5} \ln(e), \quad (14)$$

$I_{kd} =$

$$\int_{-\infty}^{\infty} dx \exp(2x) \frac{\{(x_0/d)[1 + 2\exp(x)/e + \exp(2x)] + x[1 - \exp(2x)]\}^2}{[1 + (e+1/e)\exp(x) + \exp(2x)]^4} \quad (15)$$

$$I_{kxd} = \int_{-\infty}^{\infty} dx \exp(2x) \frac{\{(x_0/d)[1 + 2\exp(x)/e + \exp(2x)] + x[1 - \exp(2x)]\} [1 + 2\exp(x)/e + \exp(2x)]}{[1 + (e+1/e)\exp(x) + \exp(2x)]^4} \quad (16)$$

Euclidean Lagrangian (with inverted potential) $L = T_k + E(x_0, d)$ makes it possible to calculate the action $S = \int L dt$ by solving the system of two Euler-Lagrange equations in the standard way.

4. ACTIVATED-TUNNELING MODE

At a low but finite temperature, both quantum and thermal fluctuations contribute to the overcoming the barrier. Often, the temperature boundary between the determining influence of one or the other is found simply by equating the probabilities of thermally activated and tunnel overcoming the barrier. However, this approach is inaccurate not only quantitatively, but also qualitatively, since it obscures the physical picture of what is happening. In fact, at a temperature other than absolute zero, an intermediate combined process is possible: tunneling not from the ground, but from the thermally excited state of the system. The probability of such the combined process is mainly determined by some optimal energy preceding tunneling which, as a rule, increases with increasing temperature. The transition temperature is considered to be the one at which the preactivation energy is compared with the barrier height E_M and above which the process has the character of the classical thermal activation.

Consider the sub-barrier motion with the preliminary activation for some energy E . The height of the barrier for the subsequent tunneling in this case decreases by the value E , and the action is a decreasing function $S(E)$ of the preactivation energy. The transition probability is equal to the product of the activation probability with the energy E , given by the Boltzmann factor $\exp(-E/kT)$, and the probability of the tunnel transition through the barrier lowered by E , which is $\exp\{-E/kT - S(E)/\hbar\}$. The optimal preactivation energy corresponds to the maximum exponent over E and is found from the equation

$$\frac{d}{dE} [E/kT + S(E)/\hbar] = 1/kT + \frac{1}{\hbar} \frac{dS}{dE} = 0. \quad (17)$$

The maximum possible solution of this equation at the preactivation energy equal to the barrier height $E = E_M$ corresponds to the transition temperature from the classical thermally activated overcoming of the barrier to its overcoming with the participation of the quantum tunneling.

5. THE TEMPERATURE OF THE TRANSITION BETWEEN MODES OF THE BARRIER OVERCOMING

To study the transition with the decreasing temperature from the classical activated jump over the barrier to the barrier overcoming with the participation of the quantum mechanical tunneling, it is necessary to study the dynamics of the system near the maximum of the potential relief E_M . Let us expand the expressions for the potential and kinetic energies by small deviations near the saddle point x_M and d_M : $x_0 \approx x_M + x$, $d \approx d_M + \delta$. The potential energy will take the form

$$E(x_0, d) \approx E_M + \frac{1}{2} K_x x^2 + K_{xd} x \delta + \frac{1}{2} K_d \delta^2. \quad (18)$$

Here

$$K_x = \frac{e_M}{d_M^2} \frac{\partial^2 E}{\partial e^2}, \quad (19)$$

$$K_{xd} = \frac{e_M}{d_M} \left\{ \frac{\partial^2 E}{\partial e \partial d} - \ln(e_M) \frac{e_M}{d_M} \frac{\partial^2 E}{\partial e^2} \right\}, \quad (20)$$

$$K_d =$$

$$\frac{\partial^2 E}{\partial d^2} - 2\ln(e_M) \frac{e_M}{d_M} \frac{\partial^2 E}{\partial edd} + \ln^2(e_M) \frac{e_M^2}{d_M^2} \frac{\partial^2 E}{\partial e^2} \quad (21)$$

The M index marks that all parameters are taken at their values at the maximum of the barrier. The kinetic energy will be given by the quadratic form

$$T_k = \frac{1}{2} M_x \dot{x}^2 + M_{xd} \dot{x} \dot{\delta} + \frac{1}{2} M_d \dot{\delta}^2. \quad (22)$$

Here the components of the anisotropic mass are

$$M_x = 2 \left\{ \frac{I_{kxM}}{2I_{1M}} E_M + \frac{I_{kdM}}{2I_{1M}} \frac{e_M^4}{E_M} (2I_{1M} + e_M \frac{dI_{1M}}{de})^2 + \frac{I_{kxdM}}{I_{1M}} e_M^2 (2I_{1M} + e_M \frac{dI_{1M}}{de}) \right\}, \quad (23)$$

$$M_{xd} = \left\{ \frac{I_{kdM}}{I_{1M}} (2I_{1M} + e_M \frac{dI_{1M}}{de}) + \frac{I_{kxdM}}{I_{1M}} E_M \right\}, \quad (24)$$

$$M_d = \frac{I_{kdM}}{I_{1M}} E_M. \quad (25)$$

Potential relief (18) near the maximum has the form of a saddle (see Fig. 2) with negative curvature along one direction (pass) y_s and positive curvature along the other y_t , transverse to the pass. The optimal trajectory to overcome the barrier corresponds to movement along the pass direction without excitation of the transverse mode. Minimizing the potential energy (18), for example, with respect to δ , one find that along the pass $\delta = -(K_{xd}/K_d)x$. Substituting this relationship into the potential and kinetic energies (18) and (22), one obtains that the movement in the pass direction is described by the Euler-Lagrange equation

$$\ddot{y}_s = -\omega^2 y_s, \quad (26)$$

where

$$\omega = \left\{ - (K_x - K_{xd}^2/K_d) / [M_x + M_d (K_{xd}/K_d)^2 - 2M_{xd} K_{xd}/K_d] \right\}^{1/2}.$$

The action for such a movement is easily calculated. Integration of equation (26) gives

$$\frac{1}{2} \dot{y}_s^2 = -\frac{1}{2} \omega^2 y_s^2 + \text{const}. \quad (27)$$

The initial conditions correspond to a start with zero velocity $\dot{y}_s = 0$ from the boundary of the classically allowed region $y_s = -y_0$ with the given preactivation energy $\frac{1}{2} \omega^2 y_0^2 = E_M - E_c = \Delta E$. Hence

$\text{const} = \Delta E = \frac{1}{2} \omega^2 y_0^2$, and equation (27) takes the form

$$\frac{1}{2} \dot{y}_s^2 = \frac{1}{2} \omega^2 (y_0^2 - y_s^2). \quad (28)$$

The action for motion during the half-period π/ω , calculated using equation (28), is

$$\begin{aligned} \frac{1}{2} S &= \int_0^{\pi/\omega} dt \left(\frac{1}{2} \dot{y}_s^2 + \Delta E - \frac{1}{2} \omega^2 y_s^2 \right) = \\ \omega \int_{-y_0}^{y_0} dy (y_0^2 - y^2)^{1/2} &= \frac{\pi}{2} (2\Delta E)/\omega. \end{aligned} \quad (29)$$

The solution to the equation for the preactivation energy (17) exists only for $T_c/T > \min|dS/dE| = 2\pi/\omega$ in accordance with the known results [14,15] for the second-order phase transition. Thus, the temperature at which the tunnel contribution appears is

$$T_c/T_q = \omega/2\pi = \frac{1}{2^{3/2}\pi} \left\{ - (K_x - K_{xd}^2/K_d) / [M_x + M_d (K_{xd}/K_d)^2 - 2M_{xd} K_{xd}/K_d] \right\}^{1/2}. \quad (30)$$

The result for the dependence of the transition temperature on the driving force f , calculated using this expression, is shown in Fig. 4. The presented picture can serve as a phase diagram for different mechanisms of the dynamics of extended quasi-one-dimensional systems. The inset shows a section of such the diagram for a Sn nanowire investigated in [16], which demonstrates a qualitative similarity with the initial section of the theoretical curve. Quantitative parameters similar to those found for numerous other materials [3] are: nanowire diameter is 20 nm $\approx (1/10)\xi$, ξ is the coherence length of bulk tin, $T_{c0} \approx 4.1$ K is the superconducting transition temperature, $I_c^{up} \approx 17$ μ A is the critical current of the wire at the lowest temperature of the experiment, 0.47 K, $I_{c0} \approx 9.5$ μ A is the current of the transition to the individual phase slip at the same temperature.

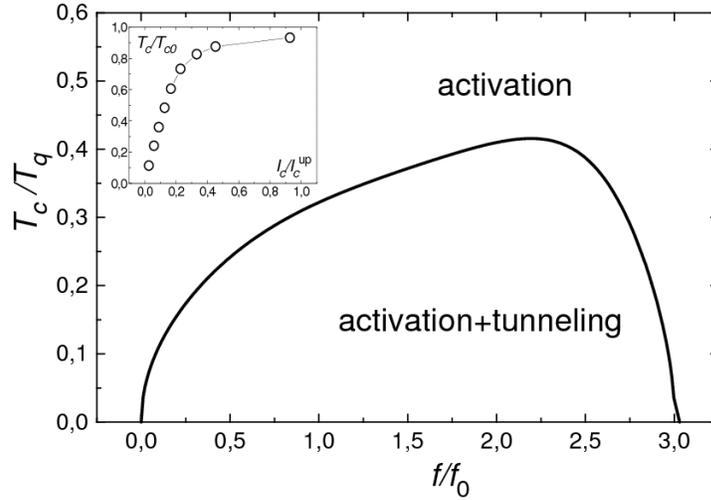


Fig. 4. The dependence on the driving force the temperature of the change of regimes between the classical thermally activated and activation-tunneling overcoming of barriers [$f_0=U_M/h$, $T_q=(\hbar/kh)(U_M/\rho)^{1/2}$]. The inset shows the experimentally observed boundary between two modes in a Sn nanowire, one of which corresponds to the multiple phase slip events due to the thermal activation, and the second corresponds to the individual phase slip due to the quantum tunneling according to the rescaled data from [16]

6. DISCUSSION AND CONCLUSION

In the present paper, the transition with the decreasing temperature from the classical thermally activated mechanism of motion of an extended system through potential barriers to motion involving the quantum tunneling is studied. The calculation was performed for any external load magnitude using the method of collective coordinates developed in the works [11,12]. A trial function qualitatively reproducing the shape of a fluctuation that transfers the system through the barrier (see Fig. 1) and is characterized by two parameters (collective coordinates) was used. It allows one to reduce the potential relief for a system with an infinite number of degrees of freedom to a two-dimensional one, as illustrated in Fig. 2. The quantitative efficiency of the method was tested by the comparing the result of calculation of the activation energy E_M for overcoming the barrier with the result of an accurate numerical calculation of this value in Fig. 3.

The rate of overcoming the barrier by the classical thermal fluctuation is mainly determined by the value of the activation energy E_M . However, with decreasing temperature, the probability of thermal fluctuations sharply decreases, and athermal quantum ones become more effective. To describe the boundary of the

appearance of a quantum contribution to overcoming the barrier, it is essential to study the potential relief near its maximum, where it has the form of a "saddle", as illustrated in Fig. 2. The barrier near the relief maximum has the shape of a parabola, the dynamics of the quantum mechanical overcoming of which is well known. This allows one to solve the problem under study.

The transition boundary can be controlled by an external load, so the main goal was a more complete calculation of the transition temperature dependence on the load than it was done before. Previously, such a transition was studied, in particular, in [17], and for the region of low values of the driving force f , the dependence of the transition temperature T_c was found $T_c \propto f^{1/2}$. Earlier in [12], it was established that for any extended (not only harmonic) potential relief, with the driving force approaching the critical value f_c which eliminates the barriers, the transition temperature behaves like $T_c \propto (f_c - f)^{1/4}$. In these two cases, the scope of the manifestation of quantum effects is rather limited. In the present paper, the dependence of the transition temperature T_c on the external load in the whole range of its variation has been calculated. The whole range includes the region in which T_c significantly increases with increasing load. This fact justifies the expansion of the scope of the

low-temperature quantum effects manifestation in the dynamics of extended systems of various nature accessible to experimental observation.

When current-carrying superconducting nanowires are used as single-photon detectors [3], an increase in the current leads to an increase in their sensitivity. At the same time, the corresponding lowering of the barrier for the escape from the metastable state increases the probability of fluctuational detector response due to background processes of phase slip and the formation of so-called “dark spots”. To find a compromise, the phase diagram of various mechanisms of the decay of metastable states of quasi-one-dimensional nanosystems calculated in this work can be useful.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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